International Astronomy and Astrophysics Competition

Pre-Final Round 2020

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article each to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 21. June 2020, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 29. June 2020.

Good luck!

Problem A.1: Interstellar Mission (4 Points)

You are on an interstellar mission from the Earth to the 8.7 light-years distant star Sirius. Your spaceship can travel with 70% the speed of light and has a cylindrical shape with a diameter of 6 m at the front surface and a length of 25 m. You have to cross the interstellar medium with an approximated density of 1 hydrogen atom/m³.

(a) Calculate the time it takes your spaceship to reach Sirius.

(b) Determine the mass of interstellar gas that collides with your spaceship during the mission.

Note: Use 1.673×10^{-27} kg as proton mass.

(a)

$$t = \frac{s}{v} = \frac{\text{distance}}{\text{velocity}} = \frac{8.7 \text{ ly}}{0.7 \frac{\text{ly}}{\text{year}}} = \boxed{12.428 \text{ years}}$$

$$\left[\because v = 0.7c = 0.7 \frac{\text{ly}}{\text{year}}\right]$$

(b)

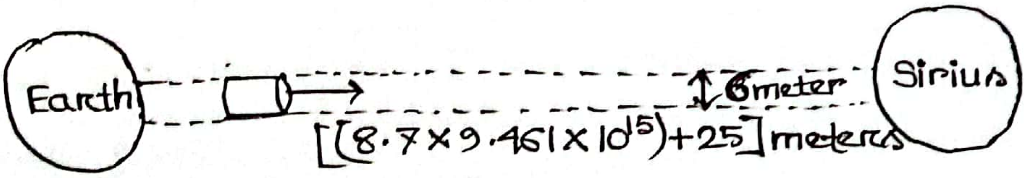
Volume of the spaceship $= \pi r^2 h = \pi \times \left(\frac{6}{2}\right)^2 \times 25 \text{ m}^3$

Volume of the whole path travelled by spaceship $= \pi \times 9 \times (25 + 8.7 \times 9.461 \times 10^{15}) \text{ m}^3$

$$M = m_p \times \rho V = m_p \times \rho \times (\pi \times 9 \times 25) \times s$$

$$= (1.673 \times 10^{-27}) \times 1 \times [(8.7 \times 9.461 \times 10^{15}) + 25] \times \pi \times 9 \text{ kg}$$

$$= \boxed{3.8935 \times 10^{-9} \text{ kg}}$$



Problem A.2: Time Dilation (4 Points)

Because you are moving with an enormous speed, your mission from the previous problem A.1 will be influenced by the effects of time dilation described by special relativity: Your spaceship launches in June 2020 and returns back to Earth directly after arriving at Sirius.

(a) How many years will have passed from your perspective?

(b) At which Earth date (year and month) will you arrive back to Earth?

(a) $t' \cancel{t_0} = \frac{8.7}{0.7} = 12.428$ years

$$t' = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow t_0 = t'\sqrt{1-\frac{v^2}{c^2}} = 12.428 \times \sqrt{1-\left(\frac{0.7c}{c}\right)^2}$$

$= 8.875$ years [time to reach Sirius]

∴ Total time passed after arrival to Sirius and returning to the earth $= (8.875 \times 2) = \boxed{17.75 \text{ years}}$

~~(b)~~

(b) Earth time passed after arrival $= (12.428 \times 2)$ years $= 24.856$ years

~~17.75 years = 17 year 9 months~~

24.856 years = 24 years 10 months 8 days

So, after 10 months, the month will be ~~March~~ April, and 1 year will be passed, meanwhile.

So, June 2020 is the starting date and the arrival date on the earth will be → $\boxed{\text{April } (2020+24+1) = 2045}$

$\boxed{\text{∴ According to earth date I will arrive back on April 2045.}}$

Problem A.3: Magnitude of Stars (4 Points)

The star Sirius has an apparent magnitude of -1.46 and appears 95-times brighter compared to the more distant star Tau Ceti, which has an absolute magnitude of 5.69.

(a) Explain the terms *apparent magnitude, absolute magnitude* and *bolometric magnitude.*
(b) Calculate the apparent magnitude of the star Tau Ceti.
(c) Find the distance between the Earth and Tau Ceti.

(a) Apparent magnitude: Apparent magnitude of a star is a value that tells how bright that star appears at its great distance from the earth. Larger magnitudes means fainter stars.

Absolute magnitude: It is the apparent magnitude the star would have if it were placed at a distance of 10 parsecs from the earth.

Bolometric magnitude: It is the measure of the total radiation of a star emitted across all wavelength of electromagnetic spectrum.

(b) Apparent magnitude of the star Tau Ceti

$$m_2 - m_1 = -2.512 \log\left(\frac{B_2}{B_1}\right)$$

$$\Rightarrow m_2 = m_1 - 2.512 \log\left(\frac{B_2}{95 B_2}\right)$$

$$= -1.46 - 2.512 \log\left(\frac{1}{95}\right)$$

$$= \boxed{3.508}$$

(Ans)

Here given,
Apparent magnitude of Sirius, $m_1 = -1.46$
$\therefore B_1 = 95 B_2$

(c) Distance between the Earth and Tau Ceti,

$$d = 10pc \times 10^{(m - M_V)/5}$$

$$= 10 \times 10^{(3.508 - 5.69)/5}$$

$$= 10 \times 10^{-0.4364}$$

$$= 3.661 pc$$

$$= (3.661 \times 3.26156)\ ly$$

$$\doteq \boxed{11.94\ ly}$$

(Ans)

Given,
$m = 3.508$
$M_V = 5.69$

Problem A.4: Emergency Landing (4 Points)

Because your spaceship has an engine failure, you crash-land with an emergency capsule at the equator of a nearby planet. The planet is very small and the surface is a desert with some stones and small rocks laying around. You need water to survive. However, water is only available at the poles of the planet. You find the following items in your emergency capsule:

- Stopwatch
- Electronic scale
- 2m yardstick
- 1 Litre oil
- Measuring cup

Describe an experiment to determine your distance to the poles by using the available items.

Hint: *As the planet is very small, you can assume the same density everywhere.*

The distance between the equator and the pole means the radius of that planet. To measure this we need to find g, ρ, M.

Let, R= Radius of planet, g= acceleration due to gravity of the planet

ρ = density of planet, M= mass of the planet.

Step-1: I will drop a stone from the top of 2m yardstick & count ~~calculate~~ the time of fall with stopwatch. When the $t=0$, then $u=0$. From the equation of motion, we can write,

$$h = \frac{1}{2} g t^2 \quad [\because \text{initial velocity, } u=0]$$

$$\Rightarrow g = \frac{2h}{t^2} = \frac{2\times 2}{t^2} = \frac{4}{t^2} \text{ --- (i)} \quad [\because h = 2m]$$

Now we know the value of 'g'.

Step-2: With the electronic scale, I will measure the weight of the stone. Here, W is the scale reading.

$\therefore$ mass of the stone, $m = \frac{W}{g}$ [$\because W$ & g are known]

Step-3: I will fill half of the measuring cup using the oil. Then, afterb dropping the stone in the measuring cup I'll notice the increase of height of oil level.

Problem-B·1

(d) We got from our previous calculation $T_E = -18.48°c$, based on this, Earth's expected average global temperature is below the freezing point of water.

But the actual averge temperature of Earth is approx. 14°c (57°F). The Earth is warmer than predicted by 32·48°c which is a big difference.

The reason of the difference of calculated and actual temperature is 'Greenhouse effect'. The primary greenhouse gases in Earth's temperature are H_2O, CO_2, CH_4, N_2O, O_3, etc. These gases in atmosphere trap some extra heat, warming our planet like a blanket. This extra warming is called 'Greenhouse effect'. Without it, our planet would be a frozen ball of ice.

So, to get an even more precise temperature estimate the effect of inconsistency of albedo of Earth due to change in the area of polar ice caps, during volcanic eruptions, due to emission of volcanic dust, human production of energy, geothermal heat, internal heat of the Earth should be considered.

Problem B.2: Distance of the Planets (6 Points)

The table below lists the average distance R to the Sun and orbital period T of the first planets:

	Distance	Orbital Period
Mercury	0.39 AU	88 days
Venus	0.72 AU	225 days
Earth	1.00 AU	365 days
Mars	1.52 AU	687 days

(a) Calculate the average distance of Mercury, Venus and Mars to the Earth. Which one of these planets is the closest to Earth on average?

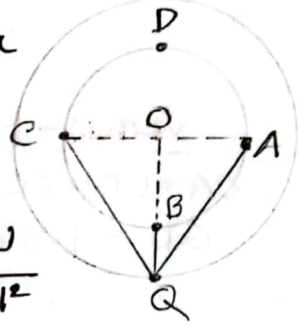
(b) Calculate the average distance of Mercury, Venus and Earth to Mars. Which one of these planets is the closest to Mars on average?

(c) What do you expect for the other planets?

Hint: *Assume circular orbits and use symmetries to make the distance calculation easier. You can approximate the average distance by using four well-chosen points on the planet's orbit.*

Mercury - Earth:

(a) Let, the inner circle be the orbit of Mercury, outer circle be the orbit of the earth. So,



Radius of inner circle $= OA = OB = OC = OD = 0.39 AU$

$OQ = 1$ AU, $\therefore BQ = 1 - 0.39 = 0.61$ AU

$DQ = OD + OQ = 1 + 0.39 = 1.39$ AU

$AQ = CQ = \sqrt{OA^2 + OQ^2} = \sqrt{(0.39)^2 + 1^2}$

$= 1.07$ AU

Average distance of the earth from the 4 points of mercury

$$d = \frac{BQ + AQ + CQ + DQ}{4} = \frac{0.61 + 1.07 + 1.07 + 1.39}{4}$$

$$= \boxed{1.035 \text{ AU}}$$

Venus - Earth: Similarly, inner orbit $\rightarrow$ Venus

outer orbit $\rightarrow$ Earth

$OA = OB = OC = OD = 0.72$ AU

$OQ = 1$ AU, $BQ = 0.28$ AU

$DQ = 1 + 0.72 = 1.72$ AU

$AQ = CQ = \sqrt{(0.72)^2 + (1)^2} = 1.23$ AU

$$d = \frac{0.28 + 1.23 + 1.23 + 1.72}{4} = \boxed{1.115 \text{ AU}}$$

Problem-B.2

Mars-Earth: Let, inner orbit → earth
outer orbit → Mars

$OA = OB = OC = OD = 1\ AU$

$OQ = 1.52\ AU$, $BQ = 0.52\ AU$

$QD = 1 + 1.52 = 2.52\ AU$; $AQ = CQ = \sqrt{1^2 + (1.52)^2} = 1.81\ AU$

$$d = \frac{0.52 + 1.81 + 1.81 + 2.52}{4} = \boxed{1.67\ AU}$$

From the average distances calculated above, it's evident that Mercury is the closest to the earth.

(b) **Mercury-Mars:** inner → Mercury, outer → Mars

$OA = OB = OC = OD = 0.39\ AU$, $OQ = 1.52\ AU$, $BQ = 1.13\ AU$

$QD = 1.52 + 0.39 = 1.91\ AU$, $AQ = CQ = \sqrt{(0.39)^2 + (1.52)^2} = 1.57\ AU$

$$d = \frac{1.13 + 1.57 + 1.57 + 1.91}{4} = \boxed{1.545\ AU}$$

Venus-Mars: inner → ~~Mercury~~ Venus, outer → Mars

$OA = OB = OC = OD = 0.72$, $OQ = 1.52$, $BQ = 0.8\ AU$

$QD = 1.52 + 0.72 = 2.24\ AU$, $AQ = CQ = \sqrt{(0.72)^2 + (1.52)^2}$

$= 1.68\ AU$

$$d = \frac{0.8 + 1.68 + 1.68 + 2.24}{4} = \boxed{1.6\ AU}$$

Earth-Mars: From (a) we get,

$$d = \boxed{1.67\ AU}$$

Here, the closest planet to the Mars will be Mercury.

(c) From Venus, the closest planet will be Mercury with average distance, $d = \frac{0.33 + 1.11 + 0.82 + 0.82}{4} = 0.77\ AU$. ~~from V~~

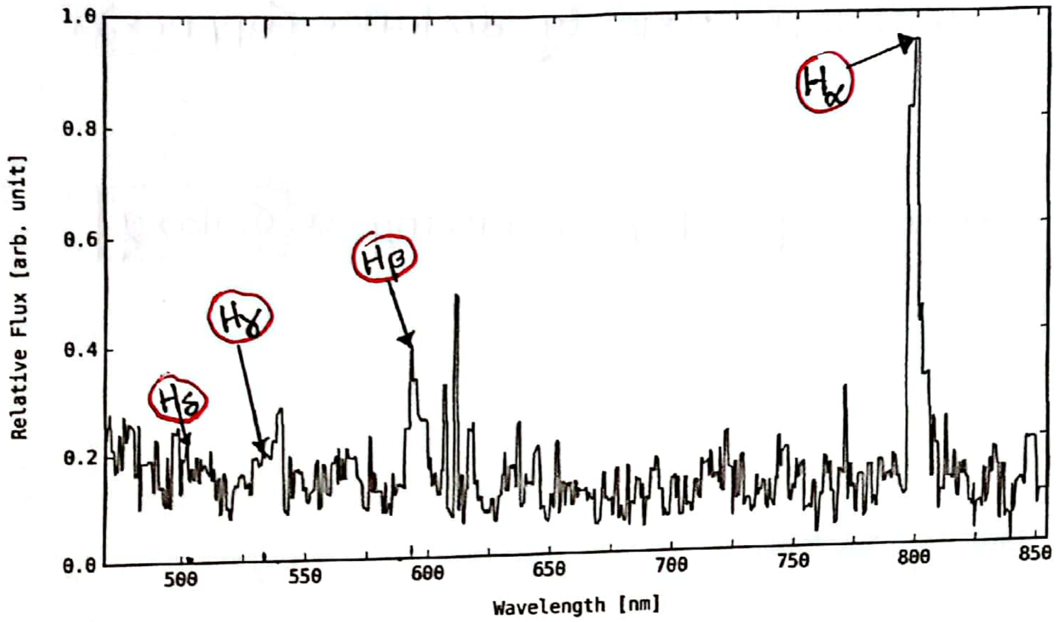
From Mercury, the closest planet will be Venus with 0.77 AU average distance.

Within the Solar System, these 4 planets are inner planets closest to the sun, so they are called "terrestitial planets". Outer planets Jupiter, Saturn, Uranus, Neptune are "Jovian/Gas planets". They've rings and lots of ~~st~~ satellites, low densities.

Terrestrial planets have relatively high densities, rapid rotation, rings, few satellites.

Problem B.3: Mysterious Object (6 Points)

Your research team analysis the light of a mysterious object in space. By using a spectrometer, you can observe the following spectrum of the object. The $H\alpha$ line peak is clearly visible:



(a) Mark the first four spectral lines of hydrogen ($H\alpha$, $H\beta$, $H\gamma$, $H\delta$) in the spectrum.
(b) Determine the radial velocity and the direction of the object's movement.
(c) Calculate the distance to the observed object.
(d) What possible type of object is your team observing?

Radial velocity $[V_r = z \times c]$

(b)

Line	λ_0 (nm)	λ_1 (nm)	z (redshift)	V_r (kms^{-1})	
$H\alpha$	656.28	800	0.21899	6.57×10^4	$\rightarrow V_r = 0.21899c \times 10^{-3}\ kms^{-1}$
$H\beta$	486.17	592	0.21768	6.53×10^4	$\rightarrow V_r = 0.21768c \times 10^{-3}\ kms^{-1}$
$H\gamma$	434.07	531	0.22330	6.69×10^4	$\rightarrow V_r = 0.22330c \times 10^{-3}\ kms^{-1}$
$H\delta$	410.17	502	0.22388	6.72×10^4	$\rightarrow V_r = 0.22388c \times 10^{-3}\ kms^{-1}$

$\therefore$ average $= 6.6275 \times 10^4\ kms^{-1}$

$[c = 3 \times 10^8\ ms^{-1}]$

The redshift is positive, so the object is moving away from us. The ~~vel~~ radial velocity is $6.6275 \times 10^4\ kms^{-1}$ (average).

$$V_r = c \times \frac{\lambda_1 - \lambda_0}{\lambda_0} = c \times \frac{\Delta\lambda}{\lambda_0}$$

(c) The distance in light-year, ~~$d_{ly} = \frac{V_r}{H_0} \times 3.26 \times 10^6$~~ $d_{ly} = \frac{V_r}{H_0} \times 3.26 \times 10^6$

$H_0 = 73.02 \left(\frac{kms^{-1}}{Mpc}\right)$

$1\ Mpc = 3.26 \times 10^6\ ly$

For $H\alpha$, $d = \frac{6.57 \times 10^4}{73.02} \times 3.26 \times 10^6\ ly$

$= \boxed{2.9332 \times 10^9\ ly}$

For $H\beta$, $d = \frac{6.53 \times 10^4}{73.02} \times 3.26 \times 10^6 = \boxed{2.9153 \times 10^9\ ly}$

Similarly,

for H_γ, $d = \cancel{2.9}\ \boxed{2.9868 \times 10^{9}\ ly}$

For H_δ, $d = \boxed{3 \times 10^{9}\ ly}$

$\therefore$ The object travelled 3×10⁹ ly distance (approx.).

(d) Our team is possibly observing a $\boxed{\text{Galaxy}}$

Problem B.4: Distribution of Dark Matter (6 Points)

The most mass of our Milky Way is contained in an inner region close to the core with radius R_0. Because the mass outside this inner region is almost constant, the density distribution can be written as following (assume a flat Milky Way with height z_0):

$$\rho(r) = \begin{cases} \rho_0, & r \le R_0 \\ 0, & r > R_0 \end{cases}$$

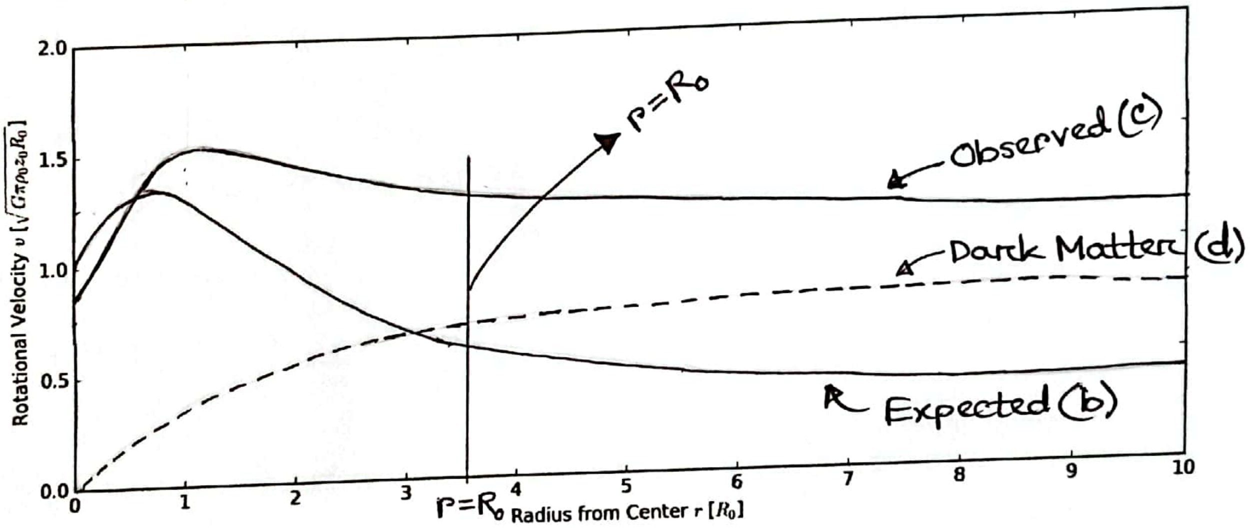
(a) Derive an expression for the mass $M(r)$ enclosed within the radius r.

(b) Derive the expected rotational velocity of the Milky Way $v(r)$ at a radius r.

(c) Astronomical observations indicate that the rotational velocity follows a different behaviour:

$$v_{obs}(r) = \sqrt{G\pi\rho_0 z_0 R_0}\left(\frac{5/2}{1+e^{-4r/R_0}} - \frac{5}{4}\right)$$

Draw the expected and observed rotational velocity into the plot below:



(d) Scientists believe the reasons for the difference to be *dark matter*: Determine the rotational velocity due to dark matter $v_{DM}(r)$ from R_0 and draw it into the plot above.

(e) Derive the dark matter mass $M_{DM}(r)$ enclosed in r and explain its distributed.

(f) Explain briefly three theories that provide explanations for *dark matter*.

(a) centripetal force = Gravitational force between two masses

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow v^2 = \frac{GM}{r}$$

$$\Rightarrow M = \frac{v^2 r}{G} = 1.5\times10^{10} \times v^2 \times r \quad \text{—— (i)}$$

Here,
$M_{sun} = 2\times10^{30}$ kg
$G = 6.673\times10^{-11}\ m^3\,kg^{-1}\,s^{-2}$

$$M = \rho_0 V \Rightarrow M(r) = \begin{cases} \rho_0 \times (\text{volume}), & r \le R_0 \\ \rho_0 \times (\text{volume}), & r > R_0 \end{cases}$$

$$\Rightarrow \boxed{M(r) = \begin{cases} \rho_0(\pi r^2 z_0), & r \le R_0 \\ 0, & r > R_0 \end{cases}}$$

$$M = 1.5\times10^{10} \times v^2 r = M_{sun} \times 2.328\times10^5 \times\left(\frac{r}{kpc}\right)\times\left(\frac{v}{km s^{-1}}\right)^2$$

$$\boxed{\therefore \frac{M(r)}{M_{sun}} = 2.328\times10^5 \times\left(\frac{r}{kpc}\right)\times\left(\frac{v}{km s^{-1}}\right)^2}$$

[$\because$ 1 kpc $= 3.086\times10^{19}$ m]

[v = circular motion velocity]

Here, v is nearly constant at most radii.

(b) We know, $\frac{GM(r)}{R_0^2} = \frac{v^2(r)}{r}$

$$v(r) = \begin{cases}\sqrt{\frac{G\pi z_0 r \rho_0}{1}}, & r\le R_0\\ \sqrt{\frac{G\pi z_0 R_0^2 \rho_0}{r}}, & r>R_0\end{cases}$$

$$\boxed{v(r) = \begin{cases}\sqrt{G\pi z_0 r \rho_0}, & r\le R_0\\ \sqrt{\frac{G\pi z_0 R_0^2 \rho_0}{r}}, & r>R_0\end{cases}}$$

We know,

$$R = R_\odot \sin l$$

$$v = v_r + v_\odot \sin l$$

$$v_r = R_\odot \sin l\left(\frac{v}{R} - \frac{v_\odot}{R_\odot}\right) = R_\odot \sin l\,(\omega - \omega_\odot)$$

[$\because v_\odot = 200$ to 238 km s^{-1}]

[$\because R_\odot = 7.6$ kpc]

v_r is the radial velocity.

Rotational velocity using radial velocity

$$v(R) = \frac{R}{R_\odot}\left(\frac{v_r}{\sin l} + v_\odot\right)$$

$$\Rightarrow v(r) = \frac{r}{R_\odot}\left(\frac{v_r}{\sin l} + v_\odot\right)$$

v = orbital velocity from GC to star/cloud

$v_\odot$ = orbital velocity from GC to sun

R = distance from GC to star/cloud

$R_\odot$ = distance from GC to sun

l = Galactic longitude of the observed star

ω = angular velocity from GC to star/cloud

(d) According to NFW model, NFW density profile

$$\cancel{\rho_{NFW}(R)} \cdot \rho_{NFW}(R) = \frac{\rho_{NFW}^0}{R/h\,/\left[1+\left(R/h\right)^2\right]}$$

circular velocity is $V_h(R) = \sqrt{\frac{GM_h(R)}{R}}$ [NFW Model]

where M_h is the enclosed mass within the scale radius h.

Here, $h = R_0$ = core radius

So, $\boxed{V_h(r) = \sqrt{\frac{GM_h(r)}{r}} \text{ according to NFW Model.}}$

According to Isothermal model

$$\rho_{iso}(r) = \frac{\rho_{iso}^0}{1+\left(\frac{r}{R_0}\right)^2}$$

$$\boxed{\text{[Isothermal Halo]} \quad V_h(r) = V_\infty\sqrt{1-\left(\frac{R_0}{r}\right)\tan^{-1}\left(\frac{r}{R_0}\right)}}$$

[$\because V_\infty^2 = 4\pi G \rho_0 R_0^2$]

(e) Dark matter mass:

$$\rho_{halo}(r) = \frac{\rho_s}{x(1+x)^2}, \quad x = \frac{r}{r_s} = \frac{r}{R_0}$$

$$\boxed{M_{halo}(r) = 4\pi\rho_s r_s^3 f(x) = M_{vir} f(x)/f(c)}$$

[$r_s = R_0$ = core radius or scale radius]

$$f(x) = \ln(1+x) - \frac{x}{1+x}.$$

[ρ_{cr} = critical density of the university]

[r_{vir} = virial radius]

$$C = \frac{r_{vir}}{r_s} = \frac{r_{vir}}{R_0} = \text{halo concentration}$$

$$M_{vir} = \frac{4\pi}{3}\rho_{cr}\Omega_0\delta_{th} r_{vir}^3 = \text{virial mass}$$

δ_c is a characteristic density

The overdensity of a collapsed object in the 'top-hat' collapse model δ_{th} (=340 in this model)

$$\left[\rho_{cr} = \frac{3H^2}{8\pi G}\right]$$

$$[\rho_s = \rho_{cr}\delta_c]$$

M_{vir} = virial mass

$$C = 15 - 3.3\log\left(\frac{M_{vir}}{10^{12}h^{-1}M_\odot}\right)$$

This dark matter forms a spherical halo whose centre is at the centre of the visibe galaxy. Its diameter is atleast 100 kpc.

(f) (i) One explanation for dark matter is that it is a property of space. Einstein realized first that empty space is not nothing. Space has amazing properties. 1st property he discovered the existence of coming more space. 2nd prediction is empty space can possess its own energy. More of this energy-of-space would appear as more space come into existence.

(ii) Another explanation for how space acquires energy comes from the quantum theory of matter. In this theory, empty space is full of temporary particles that continuously form and disappear.

(iii) 3rd explanation for dark matter is that it is a new kind of dynamical energy fluid or field, something that fills all of space but something whose effect on the expansion of the universe is the opposite of that of matter and normal energy. Some theorist named it 'quintessence'.

85% of matter of the Universe is dark.

Problem C.1 : Detection of Gravitational Waves (10 Points)

This problem requires you to read the following recently published scientific article:

Observation of Gravitational Waves from a Binary Black Hole Merger.
B. P. Abbott et al., LIGO Scientific Collaboration and Virgo Collaboration
arXiv:1602.03837, (2016). Link: https://arxiv.org/pdf/1602.03837.pdf

Answer following questions related to this article:

(a) How was the existence of gravitational waves first shown?

On February 11, 2016, scientists announced that LIGO had detected gravitational waves produced by the merger of 2 black holes more than a billion years from Earth.

(b) Which detectors exist around the world? Why did only LIGO detect GW150914?

TAMA 300, GEO 600, LIGO, CLIO, Virgo interferometer, GEO High Frequency, Advanced LIGO, Advanced Virgo, KAGRA (LCGT). To detect GW150914, LIGO detectors needed to combine astounding sensitivity with an ability to isolate real signals from sources of instrumental noise.

(c) Explain the components of the LIGO detectors.

Each LIGO detector consists of 2 arms, each 4 km long, comprising 1.2 m wide steel vacuum tubes arranged in an "L" shape, and covered by a 10 feet wide, 12 ft tall concrete shelter that protects the tubes from the environment.

(d) Describe the different sources of noise. How was their impact reduced?

Seismic noise, thermal noise, quantum noise, gas noise, charging noise, Laser noise, electronics noise, Beam jitter, oscillator noise, etc. Thermal noise is minimized by using low mechanical loss materials in the test masses & their suspensions. All components other than laser source are mounted on vibration isolation stages in ultrahigh vacuum. Using long-baseline broadband laser interferometers with potential for increased sensitivity.

(e) What indicates that the gravitational wave originated from the merger of a black hole?

Waveform analysis of GW150914 indicates that if it's astrophysical in origin, it's also a binary black hole merger. The waveform detected by LIGO detectors, matched the predictions of general relativity for a gravitational wave emanating from inward spiral and merger of a pair of blackholes, and subsequent ringdown of the single resulting black hole.

(f) Which are the methods used to search for gravitational wave signals in the detector data?

2 main methods are used. One is to measure changes induced by GW on the distances between freely moving test masses using coherent trains of electromagnetic waves. Other is to measure the deformation of large masses at their resonance frequencies induced by gravitational waves (GW). They're used in laser interferometric detectors, resonant mass detectors.

(g) How were the source parameters (mass, distance, etc.) determined from the data?

The matched-filter search is optimized for detecting signals, but it provides only approx. estimates of source parameters. To refine them, general ~~relativistic~~ relativity-based models, some of which include spin precession, and for each model perform a coherent Bayesian analysis to derive posterior distribution of source parameters.

Problem C.2 : First Image of a Black Hole (10 Points)

This problem requires you to read the following recently published scientific article:

First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole.
The Event Horizon Telescope Collaboration, arXiv:1906.11238, (2019). Link: https://arxiv.org/pdf/1906.11238.pdf

Answer following questions related to this article:

(a) Calculate the photon capture radius and the Schwarzschild radius of M87* (in AU).

Schwarzschild radius of M87*, $R_s = \frac{2GM}{c^2} = \frac{2\times 6.673\times10^{-11}\times 6.2\times10^{9} M_\odot}{(3\times10^8)^2}$

$= 5.9\times10^{-4}$ parsecs $= 1.9\times10^{15}$ cm

$\boxed{R_s = 121.696 \text{ AU}}$

Photon capture radius, $R_c = \sqrt{27}\, r_g = \sqrt{27}\, \frac{GM}{c^2}$

$R_c = \sqrt{27}\times\frac{6.67\times10^{-11}\times 6.2\times10^{9}\times M_\odot}{(3\times10^8)^2} = \boxed{316.175 \text{ AU}}$

(b) Why was it not possible for previous telescopes to take such a picture of the black hole?

The big challenge was to produce substantial images of the black hole even though they are relatively small compared to the scale of galaxies. EHT managed this by creating a network of telescopes across the globe using interferometry. To take picture of a black hole requires high resolution, seeing fine details, sensitivity, teasing out actual data from a very weak signal. ALMA enabled EHT to do both.

(c) Describe the components and functionality of the event horizon telescope.

The EHT is a VLBI experiment that directly measures 'visibilities', or Fourier components, of the radio brightness distribution on the sky. As the earth rotates, each telescope pair in the network samples many spatial frequencies. The EHT — a planet-scale array of 8 ground based radio telescopes forged through international collaboration — was designed to capture the images of a black hole. It has hydrogen maser frequency standards at all EHT sites.

(d) Explain the two algorithms used to reconstruct the image from the telescope data.

2 classes of algorithms named CLEAN & RML were used. CLEAN is an inverse-modeling approach that deconvolves the interferometer point-spread function from Fourier transformed visibilities. RML is a forward-modeling approach that searches for an image that is not only consistent with observed data but also favors specified image properties (e.g., smoothness and compactness).

(e) What parameters were required for the GRMHD simulations to generate an image?

2 parameters were required: the dimensionless spin $a_* = Jc/GM^2$, where J is spin angular momentum and M is the mass of the black hole. The 2nd parameter is the net dimensionless magnetic flux over event horizon $\phi = \Phi/(\dot{M} R_g^2)^{1/2}$, where Φ and $\dot{M}$ are magnetic flux & mass flux across the horizon respectively.

(f) Explain the physical origins of the features in Figure 3 (central dark region, ring, shadow).

The ~~assy.~~ asymmetric ring is produced by a combination of strong gravitational lensing and relativistic beaming, while the central flux depression is the observational sign of the black hole shadow. Black holes are expected to reveal a dark shadow caused by gravitational light bending & photon capture at the event horizon.

(g) How can the image resolution be increased in future observations?

Higher-resolution images can be obtained by going to a shorter wavelength, that is, 0.8 mm (345 GHz), by adding more telescopes and, in a more distant future, with space-based interferometry.